For ODE , we have the extra conditions as and

It’s easy to get that the exact solution is

First we solve for the spectral solution for expansion up to

Based on recursion relationship, we have matrix for derivation recursion relationship and matrix for non-constant coefficients.

Thus the matrix realization for the ODE is

Matrix truncated at is:

0 0 0 0

1 0 0 0

0 3 0 0

1 0 5 0

Matrix truncated at is:

0 1 0 0

1/3 0 2/3 0

0 2/5 0 3/5

0 0 3/7 0

Thus,

1 0 1 0

0 1 0 9

0 0 3 0

0 0 0 7

The elements of column *b* are found from

for n=3 is:

1.614643504944718

1.777899426251808

1.330606497348779

0.548143758894128

Now we put in the auxiliary conditions to replace the last two rows of with the equivalent restrictions on the coefficients:

1 0 1 0

0 1 0 9

1 0 -1/2 0

1 1 1 1

1.614643504944718

1.777899426251808

1.000000000000000

2.718281828459046

Solve for , we get:

1.204881168314906

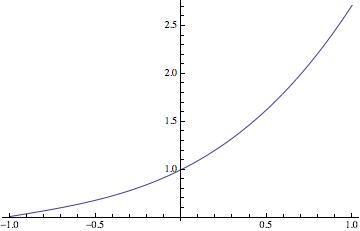
1.019355685672142

0.409762336629812

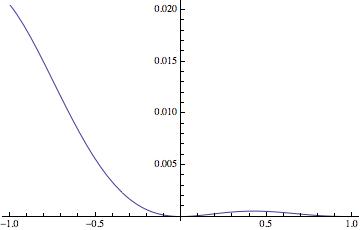
0.084282637842185

So we can expand

Plot for :



Plot for :



Secondly, we solve for the spectral solution for expansion up to

Matrix truncated at is:

0 0 0 0 0 0

1 0 0 0 0 0

0 3 0 0 0 0

1 0 5 0 0 0

0 3 0 7 0 0

1 0 5 0 9 0

Matrix truncated at is:

0 1 0 0 0 0

1/3 0 2/3 0 0 0

0 2/5 0 3/5 0 0

0 0 3/7 0 4/7 0

0 0 0 4/9 0 5/9

0 0 0 0 5/11 0

Thus,

1 0 1 0 8 0

0 1 0 9 0 36

0 0 3 0 25 0

0 0 0 7 0 49

0 0 0 0 13 0

0 0 0 0 0 21

The elements of column *b* are found from

for n=5 is:

1.614643504944718

1.777899426251808

1.330606497348779

0.548143758894128

0.137712471330815

0.019654844320030

Now we put in the auxiliary conditions to replace the last two rows of with the equivalent restrictions on the coefficients:

1 0 1 0 8 0

0 1 0 9 0 36

0 0 3 0 25 0

0 0 0 7 0 49

1 0 -0.5 0 0.375 0

1 1 1 1 1 1

1.614643504944718

1.777899426251808

1.330606497348779

0.548143758894128

1.000000000000000

2.718281828459046

Solve for , we get:

1.174571919929633

1.105197440811793

0.356937646586907

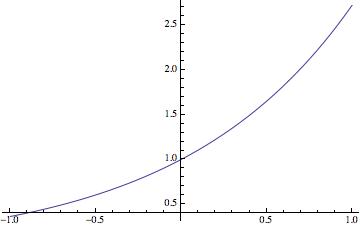
0.069995883419958

0.010391742303522

0.001187195407233

So we can expand

Plot for :



Plot for :

